Solving Equations-The Basics

An **Equation** is a mathematical statement that says one expression is equal to another expression.

To **Solve** an equation means to find all values for the variable that make the equation true.

Example: Solve the equation $x - 5 = 12$

The only value for x that makes this equation true is $= 17$, and thus 17 is the solution to the equation.

The following properties will be used to solve equations:

Addition and Subtraction Property of Equality: If $a = b$ is a given equation, then $a + c = b + c$, and $a - c = b - c$ for any real number c. In other words, you can add or subtract any real number to both sides of an equation.

Example: Solve $x - 5 = 12$ We can add 5 to both sides of the equation.

$x - 5 + 5 = 12 + 5$

$x = 17$ Which is the solution because $17 - 5 = 12$

Example: Solve $x + 6 = 22$ We can subtract 6 from both sides of the equation.

$x + 6 - 6 = 22 - 6$

$x = 16$ Which is the solution because $16 + 6 = 22$

Multiplication and Division Property of Equality: If $a = b$ is the given equation, then $ca = cb$ and $\frac{a}{c} = \frac{b}{c}$ for any real number c. In other words, you can multiply or divide any real number to both sides of an equation.

Example: Solve $3x = 15$ We can divide by 3 on both sides of the equation.

$\frac{3x}{3} = \frac{15}{3}$ (Notice that division is often written as a fraction)

$x = 5$ Which is the solution because $3 \cdot 5 = 15$

Example: Solve $\frac{x}{8} = 4$ We can multiply by 8 on both sides of the equation.

$8\left(\frac{x}{8}\right) = 4 \cdot 8$

$x = 24$ Which is the solution because $\frac{24}{8} = 4$

Use Opposite Operations:

-When the equation has an addition, then subtract to solve. When the equation has a subtraction, then add to solve. Addition and subtraction are opposites of each other.

-When the equation has a multiplication, then divide to solve. When the equation has a division, then multiply to solve. Multiplication and division are opposites of each other.

Using both properties: When solving equations, always use the opposite operation. If the equation has multiple operations, apply the Addition and Subtraction Property and then apply the Multiplication and Division Property. According to order of operations, multiplication and division are computed before addition and subtraction, but since we are solving using the opposite operations, we are also going to apply those in the opposite order. Thus in most equations, addition and subtraction are done before multiplication and division.

Example: Solve $3x + 5 = 26$ First subtract 5 from both sides of the equation.

$3x + 5 - 5 = 26 - 5$

$3x = 21$ Divide by 3.

$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

Example: $-5 + 4x = 27$ First it is easier to see the equation when the variable term is first

$4x - 5 = 27$ We now have a subtraction, and thus we add 5 to both sides of the equation.

$4x = 32$ Divide by 4.

$x = 8$

Example: $10 - 5x = 45$ First it is easier to see the equation when the variable term is first

$-5x + 10 = 45$ Notice we now have an addition, when it first looked like subtraction

$-5x = 35$ Subtract 10 from both sides of the equation

$\frac{-5x}{-5} = \frac{35}{-5}$ Divide both sides by -5

$x = -7$

Variables on Both Sides:

When variables are on both sides of the equation, use the Addition Property of Equality to add or subtract a variable term to "*move*" the variable term to the other side of the equation.

Example: $3x + 4 = 5x - 6$ Subtract 3x from both sides

$3x - 3x + 4 = 5x - 3x - 6$

$4 = 2x - 6$ Add 6 to both sides

$10 = 2x$ Divide by 2

$\frac{10}{2} = \frac{(2x)}{2}$

$5 = x$